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Identification and Semiparametric Estimation of Equilibrium Models of Local Jurisdictions†

By Dennis Epple, Michael Peress, and Holger Sieg*

We develop a new model of household sorting in a system of residential neighborhoods. We show that this model is partially identified without imposing parametric restrictions on the distribution of unobserved tastes for neighborhood quality and the shape of the indirect utility function. The proof of identification is constructive and can be used to derive a new semiparameteric estimator. Our empirical application focuses on residential choices in the Pittsburgh metropolitan area. We find that sorting of households with children exhibit more stratification by income than sorting of households without children. (*JEL* C51, D12, H41, J12, R21, R23)

Research in urban and public economics has focused on improving our understanding of the impact of local public goods and amenities on equilibrium sorting patterns of households.[1] These models take as their starting point the idea that households are at least potentially mobile. Communities differ according to their levels of public good provision, tax rates, and local housing market conditions. Each household takes these factors into account in choosing a community. If local public good provision is decentralized via local majority rule then the level of public goods will depend on characteristics of the community's residents. Households will sort among communities according to tastes and income, so that households with similar preferences for local public goods will tend to live in the same community. Since the population of each community is endogenous, the set of households living in the

* Epple: Tepper School of Business, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, PA 15213 (e-mail: epple@andrew.cmu.edu); Peress: University of Rochester, Department of Political Science, Rochester, NY 14627-0146 (e-mail: mperess@mail.rochester.edu); Sieg: Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia, PA 19104 (e-mail: holgers@econ.upenn.edu). We would like to thank an anonymous referee, Pat Bajari, Pat Bayer, Lanier Benkhard, Steve Berry, Herman Bierens, Martin Browning, Jan Bruckner, Andrew Chesher, Phil Haile, James Heckman, Lars Nesheim, Charles Manski, Rosa Matzkin, Antonio Merlo, Ariel Pakes, Richard Romano, John Rust, Matt Shum, Chris Taber, Quang Voung, Ed Vytlacil, and participants of seminars at various conferences and universities for comments and suggestions. Financial support for this research is provided by the National Science Foundation under grants SBR-0111630 and SBR-0617844, the MacArthur Foundation, and the Alfred P. Sloan Foundation. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation, the MacArthur Foundation, or the Alfred P. Sloan Foundation.

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[1] This literature was inspired by Charles Tiebout (1956). See, for example, Epple, Radu Filimon, and Thomas Romer (1984); Timothy J. Goodspeed (1989); Epple and Romer (1991); Thomas J. Nechyba (1997); Roland Benabou (1996); Raquel Fernandez and Richard Rogerson (1998); Nechyba (2000); Benabou (2002); and Jesse M. Rothstein (2006). Parallel to the development of locational equilibrium models, there has been much progress in theoretical and empirical research that analyzes spatial sorting of households in cities. This literature starts with the classic papers by William Alonso (1964); Edwin S. Mills (1967); and Richard M. Muth (1969).

community and the decisive voters in the community are jointly determined in equilibrium. When voting, and thereby collectively determining the level of public good provision within a community, voters take into consideration the interaction among housing market equilibrium, mobility, and public good provision.

An important insight from these models is that plausible single-crossing assumptions about preferences generate strong predictions about the equilibrium distribution of households across communities and neighborhoods. More recently, research has focused on devising empirical strategies, which can be used to estimate the parameters of these models, evaluate their goodness of fit, and conduct policy analysis.[2] We develop a new sorting model that uses mixtures of distributions to characterize observed and unobserved heterogeneity among households. This approach allows us to model differences in discrete as well as continuous characteristics of households. The resulting equilibrium sorting model can, therefore, be viewed as a mixture of hierarchical models of the type considered in Epple and Sieg (1999).

We then adopt a nonparametric approach to study identification in our model. We allow households to differ in their incomes, their tastes for neighborhood quality, and various other characteristics such as family structure. Since the distribution of tastes for neighborhood quality is inherently unobservable, we do not impose functional form assumptions on the distribution of tastes.[3] One objective of the analysis is then to derive general conditions that allow us to nonparametrically identify the distribution of household characteristics and the indirect utility function of households based on the observed equilibrium outcomes.

We first consider the case in which the utility function is known. We show that the discreteness of the choice set imposes limits to identification.[4] We find that it is possible to nonparametrically identify a finite number of points of the distribution of tastes conditional on income for each household type. These points correspond to the points on the boundary between adjacent neighborhoods. For points that are not on the boundary loci, we can only provide lower and upper bounds for the distribution. These bounds become tighter as the number of differentiated neighborhoods in the application increases. Joint nonparametric or semiparametric identification of the distribution of household characteristics and the indirect utility function is more difficult to establish. We show that we can provide bounds for the indirect utility function and the distribution of tastes. We thus conclude that the model is partially nonparametrically identified.[5]

[2] Parametric versions of the models considered in this paper have provided new insights into household behavior as discussed, for example, in Epple and Sieg (1999); Epple, Romer, and Sieg (2001); JunJie Wu and Seong-Hoon Cho (2003); Sieg et al. (2004); Stephen Calabrese et al. (2006); Maria Marta Ferreyra (2007); Walsh (2007); and Epple, Brett Gordon, and Sieg (2010a). See also Lars Patrick Nesheim (2001); Patrick Bajari and Matthew E. Kahn (2005); Patrick Bayer, Robert McMillan, and Kim Rueben (2004); Bayer, Fernando Ferreira, and McMillan (2007); and Ferreira (2009) for related empirical approaches which are based on more traditional discrete choice models or hedonic frameworks.

[3] Our research is thus similar to recent work by Ivar Ekeland, James J. Heckman, and Nesheim (2004); Heckman, Rosa Matzkin, and Nesheim (2005); and Bajari and C. Lanier Benkard (2005) on identification and estimation of hedonic models. It is also closely related to Richard W. Blundell, Martin Browning, and Ian A. Crawford (2003) who discuss nonparametric tests of revealed preference models.

[4] Point identification cannot be achieved in many econometric applications. In that case, attention naturally shifts to characterizing informative bounds on the parameters of interest (Charles F. Manski 1997).

[5] For a general discussion of partial identification see Guido W. Imbens and Manski (2004) and Victor Chernozhukov, Imbens, and Whitney Newey (2007).

Our proofs of identification are constructive and give rise to algorithms for estimating the functions of interest or placing bounds on important parameters. Our sorting model implies that the (unobserved) quality of the neighborhoods should be monotonically increasing in price or the price rank of the neighborhood. Moreover, this function must also have a sufficient degree of curvature to guarantee that the differences in qualities are large enough given the differences in housing prices. We can nonparametrically estimate a function which links the observed measure of quality of the neighborhood to the observed price rank of the neighborhood using recent innovations in nonparametric estimation which impose monotonicity and curvature constraints on the underlying function.[6] We then derive a testing procedure which allows us to determine which set of parameter values of the indirect utility functions are admissible, i.e., we test for which parameters the shape restrictions are valid. For each admissible indirect utility function, we can then estimate the joint distribution of income and tastes.[7]

The demand side of a locational equilibrium model shares many similarities with demand models for differentiated products in industrial organization.[8] Following Daniel McFadden (1974), most demand models for differentiated products have adopted a discrete choice approach assuming that consumers purchase only one unit of a differentiated product. While this is a reasonable assumption in many circumstances, there are a number of interesting applications in which consumers also make a quantity choice in addition to a quality choice. Consumers often face a trade-off between purchasing small quantities of a high quality and expensive product and larger quantities of a lower quality and hence cheaper product. Suitable econometric models then need to capture the discrete choice of product or neighborhood quality and the continuous choice associated with the quantity choice (housing). Our approach is thus closely related to earlier work by Jeffrey A. Dubin and McFadden (1984) and W. Michael Hanemann (1984).

A key simplifying assumption of our approach is that the desirability of a product relative to the other products available in the market can be measured by a one-dimensional index. This assumption gives rise to a model of vertical product differentiation.[9] A more general approach also allows for horizontal differentiation since consumers may have heterogeneous tastes with respect to product or neighborhood characteristics. One prominent approach in industrial organization is due to Berry (1994) and Berry, James Levinsohn, and Ariel Pakes (1995). Bayer, McMillan, and Rueben (2004) have applied these methods to estimate a model of racial sorting and peer effects using restricted use micro data from the US census. In contrast to that approach, our approach allows for more limited patterns of substitution among communities, but provides a more flexible demand system for housing services. As a consequence, our approach also provides a tractable empirical framework for analyzing voting behavior and collective choices as discussed in Epple, Romer, and Sieg (2001).

[6] See Matzkin (1994) and Arthur Lewbel and Oliver Linton (2003) for a discussion of nonparametric estimators that impose shape restrictions.

[7] Our estimation approach thus differs significantly from share inversion estimators discussed in Steven T. Berry (1994).

[8] The pioneering early papers on characteristics models with horizontal product differentiation are due to W. M. Gorman (1980) and Kevin J. Lancaster (1966).

[9] Timothy F. Bresnahan (1987) considers a discrete choice model with vertical product differentiation.

The empirical analysis focuses on residential and housing choices in the Pittsburgh metropolitan area. Our framework allows neighborhoods to differ in quality (local public goods such as public education and protection from crime), and households may choose to purchase different quantities of housing in each neighborhood. We find that there are significant differences in the observed sorting of households with and without children. In particular, households with children exhibit more stratification by income than households without children. Low-income households with children have lower tastes for local public goods and amenities than similar households without children. The opposite is true for high-income households. Households with children choose to reside in larger homes in cheaper neighborhoods.

The rest of the paper is organized as follows. Section I discusses the household demand model. Section II discusses identification. Section III develops a new semiparametric estimator motivated by our identification results. Section IV discusses the data used in this paper. Section V presents the main empirical findings. Section VI offers some conclusions and discusses future research.

I. Household Sorting and Residential Choices

We consider a metropolitan area with a finite number of household types that differ in their endowed income, y, and in a taste parameter, α, which reflects the household's strength of preferences for neighborhood quality. Each household type i captures some observed discrete differences in the population, such as family structure, and occurs with probability P_i. The continuum of households conditional on type i is implicitly described by the joint distribution of α and y, denoted by $F_i(\alpha, y)$.

ASSUMPTION 1: *The joint distribution of income and tastes $F_i(\alpha, y)$ is continuous with support $S \subseteq R^2_+$ and joint density $f_i(\alpha, y)$, for $i = 1, \ldots, I$.*

A household of type i with taste parameter α and income y is referred to as a triple (α, y, i).

A household has preferences defined over the quality of the neighborhood, g, the quantity of housing consumed, q, and a composite private good, b.[10]

ASSUMPTION 2: *The preferences of a household are represented by a utility function, $U(\alpha, g, q, b)$ that is twice differentiable in its arguments and strictly quasiconcave in g, q, and b.*

Conditional on choosing a neighborhood, thereby choosing g, the household determines the optimal amount of housing q by choosing

$$\max_{(q,b)} U(\alpha, g, q, b) \tag{1}$$

$$s.t.\ p\, q = y - b,$$

[10] We are thus assuming that households have the same utility function conditional on tastes, i.e., $U_i = U$ for all i. It is straightforward to extend the analysis in this paper to allow for differences in $U_i(\cdot)$.

where p denotes the gross-of tax price of housing. There are J different neighborhoods in the metropolitan area. Without loss of generality, we assume that $g_1 < \ldots < g_J$, which implies that $p_1 < \ldots < p_J$.

It is convenient to represent the preferences of a household that lives in neighborhood j using the indirect utility function, $V(\alpha, y, g_j, p_j)$:

$$V(\alpha, g_j, p_j, y) = U(\alpha, g_j, q(p_j, y, \alpha), y - p_j q(p_j, y, \alpha)).$$

Households choose the neighborhood that maximizes (indirect) utility:

$$\max_{d_1, \ldots, d_J} \sum_{j=1}^{J} d_j \, V(\alpha, g_j, p_j, y)$$

subject to the constraint that $\sum_{j=1}^{J} d_j = 1$ and $d_j \in \{0,1\}$. Note that the "error term" of the model (α) enters into the indirect utility function in a nonadditively separable way. As such, the model does not give rise to a standard random utility model with additive error terms.[11]

To characterize the sorting of households in this model, it is useful to impose additional assumptions on the indirect utility function. Consider the slope of an "indirect indifference curve" in the (g_j, p_j)-plane:

$$M(\alpha, y, g_j, p_j) = \left.\frac{dp_j}{dg_j}\right|_{V=\overline{V}}. \tag{2}$$

We assume that the indirect utility function satisfies standard single-crossing conditions in the (g_j, p_j)-plane.

ASSUMPTION 3: *For given* α, $M(\cdot)$ *is monotonically increasing in* y. *For given* y, $M(\cdot)$ *is strictly monotonically increasing in* α.

Note that nonadditive separability of α is a necessary (but not sufficient) condition for Assumption 3 to hold.

Let C_j denote the set of households that choose neighborhood j:[12]

$$C_j = \{(\alpha, y) \mid V(\alpha, y, g_j, p_j) \geq \max_{i \neq j} V(\alpha, y, g_i, p_i)\}. \tag{3}$$

The share of households of type i that live in neighborhood j is given by

$$n_{ij} = \int_{C_j} f_i(\alpha, y) \, d\alpha \, dy. \tag{4}$$

[11] The structure of the discrete part of the choice model is similar to a nonparametric ordered probit model. Aaron K. Han (1987) and Shakeeb Khan (2001) allow the error term to enter as a separate argument into an unknown parametric function.

[12] The set of households that are indifferent have measure zero.

Summing over all discrete types yields the total size of neighborhood j:

$$n_j = \sum_{i=1}^{I} n_{ij} P_i. \tag{5}$$

Consider an allocation in which each neighborhood has a positive size. For such an allocation to be an equilibrium, there must be an ordering of neighborhoods, $\{(g_1, p_1), \ldots, (g_J, p_J)\}$, with $g_1 < \cdots < g_J$ such that:

- **Boundary Indifference:** There exists a set of households that are indifferent between two "adjacent" neighborhoods. This set is characterized by the following expression:

$$R_j = \left\{(\alpha, y) \mid V(\alpha, g_j, p_j, y) = V(\alpha, g_{j+1}, p_{j+1}, y)\right\} \quad j = 1, \ldots, J-1. \tag{6}$$

- **Stratification:** Let $\alpha_j(y)$ be the implicit function defined by equation (6).[13] Then, for each level of income y, the households that live in j consist of those with tastes, α, given by:

$$\alpha_{j-1}(y) \leq \alpha \leq \alpha_j(y). \tag{7}$$

- **Ascending Bundles:** Consider two neighborhoods i and j, such that $p_i > p_j$. Then $g_i > g_j$ if and only if $\alpha_i(y) > \alpha_j(y)$. In particular, boundary indifference loci do not intersect.

As we will see below, these necessary conditions of equilibrium are important in establishing identification of the model.[14]

Most of the previous empirical literature in urban economics also assumes that the indirect utility function satisfies the following separability assumption:[15]

ASSUMPTION 4: *The indirect utility function is additively separable and hence can be written as:*

$$V(\alpha, y, g, p) = \alpha V^g(g) + V^b(y, p). \tag{8}$$

Using Roy's Identity, the demand functions are given by

$$q(p, y) = -\frac{\partial V^b / \partial p}{\partial V^b / \partial y}. \tag{9}$$

[13] Define $\alpha_0(y) = 0$ and $\alpha_J(y) = \infty$.

[14] A formal proof of the results above are given in Epple and Sieg (1999).

[15] This assumption is also invoked by almost all computational general equilibrium studies such as Epple and Romer (1991), Fernandez and Rogerson (1996), Nechyba (1997), Kurt Schmidheiny (2006), and Rothstein (2006).

A direct consequence of Assumption 4 is that the demand function above does not depend on α and g.[16]

Separability does not imply that the unconditional demand for housing does not depend on neighborhood characteristics. It just means that, conditional on neighborhood choice, neighborhood characteristics only affect the demand for housing if these neighborhood amenities are capitalized in the housing prices. This fact is largely supported by the empirical literature. Separability rules out a household choosing to buy a larger house just because it happens to live in a high-crime neighborhood. Almost all previous empirical and computational studies of local public good provision and residential sorting in local housing markets are either implicitly or explicitly based on this type of separability assumption.

II. Identification

The nature of the identification problem is to determine whether it is possible to identify the indirect utility function $V_0(\alpha, y, g, p)$ and the joint distributions of income and tastes $\{F_{i0}(\alpha, y)\}_{i=1}^{I}$ given the observed outcomes. Identification depends largely on the information set that is available to the econometrician. We assume that the econometrician observes the following outcomes:

ASSUMPTION 5: *For every neighborhood the econometrician observes:*

(i) *the share of households of type i that live in neighborhood j* n_{ij}, $i = 1, \ldots, I$,

(ii) *the joint density of income and housing of each household type* $i = 1, \ldots, I$,

as well as prices, p_j *and neighborhood qualities* g_j.

We thus consider identification and estimation of this class of models based on aggregate or at least grouped data. These types of data are typically available from the US census and other publicly available sources.[17] We also follow Ekeland, Heckman, and Nesheim (2004) and restrict attention to identification based on variation in one metropolitan housing market.[18]

First we consider the case in which $V(\cdot)$ is known to the econometrician.[19] Notice that knowledge of $V(\cdot)$ implies that the econometrician knows the boundary indifference loci $\alpha_j(y)$. The first result states that we can identify $J - 1$ points of the conditional distribution of $F_i(\alpha \mid y)$ for each household type.

[16] Separability of the indirect utility function is admittedly a strong assumption. Dubin and McFadden (1984) consider a parametric model with nonseparable preferences.

[17] For example, individual level data on residential choices at the local level are only available through census research centers.

[18] If we observe equilibria of the same market at successive points of time, or if we observe multiple markets at one point of time, then additional sources for identification are possible. These approaches typically impose restrictions of how preferences are allowed to vary across markets. Some results for this case are available upon request from the authors.

[19] With a slight abuse of notation, we sometimes suppress the subscript 0 that denotes the model under which the data are generated. Similarly, we do not use different symbols for marginal and joint distributions.

PROPOSITION 1: *If the indirect utility function is known, one can identify* $J - 1$ *points of* $F_i(\alpha \,|\, y)$. *These points correspond to the values of* α *implied by the* $J - 1$ *boundary indifference loci, and are given by* $\alpha = \alpha_j(y)$.

PROOF:

Note that the joint density of (α, y) of household type i that lives in neighborhood j is given by

$$f_{ij}(\alpha, y) = \begin{cases} \dfrac{f_i(\alpha, y)}{n_{ij}} & \text{if} \quad (\alpha, y) \in C_j \\ 0 & \text{if} \quad (\alpha, y) \notin C_j \end{cases} . \tag{10}$$

Hence, the marginal density of income of households of type i that lives in j is given by[20]

$$\begin{aligned} f_{ij}(y) &= \int_{C_j} f_{ij}(\alpha, y)\, d\alpha \\ &= \frac{f_i(y)}{n_{ij}} \int_{\alpha_{j-1}(y)}^{\alpha_j(y)} f_i(\alpha \,|\, y)\, d\alpha \\ &= \frac{f_i(y)}{n_{ij}} \Big[F_i(\alpha_j(y) \,|\, y) - F_i(\alpha_{j-1}(y) \,|\, y) \Big]. \end{aligned} \tag{11}$$

Rearranging terms, such that observables are on the right-hand side of the equation, yields for the first neighborhood,

$$F_i(\alpha_1(y) \,|\, y) = \frac{f_{i1}(y)}{f_i(y)}\, n_{i1} \tag{12}$$

and for all other neighborhoods $j > 1$:

$$F_i(\alpha_j(y) \,|\, y) = \frac{\sum_{k=1}^{j} n_{ik} f_{ik}(y)}{f_i(y)}. \tag{13}$$

The right-hand side of equations (12) and (13) are observed in the data by Assumption 5. Hence, we can identify $J - 1$ points of the conditional distribution function of α given y for each type i. These points correspond to the values of $\alpha_j(y)$, $j = 1, 2, \ldots, J - 1$.

[20] We assume that the distribution of income and tastes is defined on the positive quadrant $R_+ \times R_+$. Moreover, boundary loci between adjacent communities do not intersect. As a consequence the supports of the income neighborhood distributions are the same for all neighborhoods.

The results in Proposition 1 are similar to results found in the econometric literature that considers nonparametric estimation of ordered probit models and other semi-parametric discrete choice models.[21] The main difference here is that we consider identification based on aggregate data that typically are publicly available to researchers in urban economics. But as in the semiparametric discrete choice literature, we find that the discreteness of the choice set implies that we can arbitrarily transform the distribution of α given y on the intervals $(\alpha_{j-1}(y), \alpha_j(y))$ without affecting the sorting of households among neighborhoods in equilibrium, as long as the transformed distribution has the correct values at the boundaries. As a consequence, the conditional distribution of tastes given income is not identified in the interior of these intervals.

While we do not obtain point identification of $F_i(\alpha \,|\, y)$ for points which are not on the boundary loci of the model, the monotonicity of the distribution function allows us to construct bounds for these function values. Let $\underline{F}_i(\alpha \,|\, y)$ $(\overline{F}_i(\alpha \,|\, y))$ denote the lower (upper) bound. For any value of α, such that $\alpha_j(y) < \alpha < \alpha_{j+1}(y)$, we then obtain

$$\underline{F}_i(\alpha \,|\, y) \;=\; F_i(\alpha_j(y) \,|\, y) \;\leq\; F_i(\alpha \,|\, y) \;\leq\; F_i(\alpha_{j+1}(y) \,|\, y) \;=\; \overline{F}_i(\alpha \,|\, y). \tag{14}$$

If there are many neighborhoods, each with a small size, we would expect that the difference between $\alpha_j(y)$ and $\alpha_{j+1}(y)$ will be small for most adjacent neighborhoods. We thus conclude that the bounds for the conditional distribution of tastes are likely to be informative in applications with large choice sets.

We have assumed that $V(\cdot)$ is known to the econometrician. We now consider the problem of jointly identifying $V(\cdot)$ and $\{F_i(\cdot)\}_{i=1}^{I}$. Assumption 5 implies that we observe the joint distribution of income and housing for each neighborhood. If the function $V^b(y,p)$ satisfies standard integrability conditions, identification of $V^b(y,p)$ is limited only by the fact that we observe a finite number of neighborhoods. Identification of $V^g(g)$ is more problematic. Consider a sub-utility function $V^g(g)$ that yields boundary indifference loci that do not intersect, i.e., that satisfies

$$\alpha_{j+1}(y) \;>\; \alpha_j(y) \;\; \forall_j. \tag{15}$$

Any such sub-utility function is then consistent with the observed outcomes. As a consequence, we have the following result:

PROPOSITION 2: *For any sub-utility functions* $V^b(p,y)$ *and* $V^g(g)$, *such that*

(i) $V(\alpha, y, g, p) = \alpha V^g(g) + V^b(p, y)$ *satisfies assumptions 2, 3, and 4;*

(ii) $V(\alpha, y, g, p)$ *implies the same housing demand function as the true indirect utility function* $V_0(\alpha, y, g, p)$;

[21] See James L. Powell (1994) for a survey of the semi-parametric literature.

(*iii*) *boundary loci do not intersect, i.e.,* $\alpha_{j+1}(y) > \alpha_j(y)\ \forall j$, *there exist a set of distribution* $\{F_i(\alpha,y)\}_{i=1}^I$, *such that the observed sorting of households is identical to the one obtained for the true model* $V_0(\alpha, y, g, p)$ *and* $\{F_{i0}(\alpha, y)\}_{i=1}^I$.

PROOF:

Consider an indirect utility function $V_a(\alpha, y, g, p)$ that satisfies condition (i). Let $\alpha_j^0(y)$ and $\alpha_j^a(y)$ denote the boundary indifference loci that correspond to $V_0(\cdot)$ and $V_a(\cdot)$, respectively. Condition (iii) implies that

$$\alpha_{j+1}^a(y) \ > \ \alpha_j^a(y) \ \ \forall j. \tag{16}$$

Define the conditional distribution of α, $F_{ia}(\alpha\,|\,y)$ for the relevant points on the boundary loci as follows:

$$F_{ia}\big(\alpha_j^a(y)\,|\,y\big) \ \equiv \ F_{i0}\big(\alpha_j^0(y)\,|\,y\big) \quad j \ = \ 1,\ldots,J. \tag{17}$$

Then, by construction, the observed equilibrium sorting of households by income within and among neighborhoods for V_a and F_{ia} is observationally equivalent to the one given by V_0 and F_{i0}. By condition (ii), the implied joint distribution of income and quantity are also the same.

The indirect utility function is not fully identified for three reasons. First, the number of neighborhoods is finite. Hence, there are only a finite number of observations of p. Second, for those fixed values of p, the housing demand function $q(p, y)$ can only be traced out in the y dimension. Hence, we can only identify J Engel curves for different levels of p. The dependence of q on p can only be bounded for values of p that are not observed in the data. Hence, neither $q(\cdot)$ nor $V^b(\cdot)$ are fully identified. As the number of neighborhoods goes to infinity and market shares get arbitrarily small, these restrictions become unimportant. Third, $V^g(\cdot)$ is only identified from the conditions that state that the $J - 1$ boundaries cannot intersect given the observed levels of p and g. As a consequence, the demand model considered above is only partially identified.

To obtain stronger results, we adopt a semiparametric framework and introduce a parametrization of the indirect utility function while remaining as flexible as possible regarding the distribution of (α, y). For concreteness, we also adopt a form for $V^b(y,p)$ that implies constant price and income demand elasticities.

ASSUMPTION 6: *The utility function is known up to a finite vector of parameters,* θ, *and takes the form*

$$V(\alpha, y, g_j, p_j) \ = \ \left\{\alpha \ g_j^\rho \ + \left[e^{\frac{y^{1-\nu}\,-\,1}{1\,-\nu}} \ e^{-\frac{Bp_j^{\eta+1}\,-\,1}{1\,+\,\eta}}\right]^\rho\right\}^{\frac{1}{\rho}}, \tag{18}$$

where $\theta = (\rho, \eta, \nu, B)$ *and* $\rho < 0$, $\eta < 0$, $\nu > 0$, *and* $B > 0$.

Assumption 6 implies that the set of households that are indifferent between adjacent neighborhoods is characterized by the following equation:

$$\alpha_j(y) = \left[e^{\frac{y^{1-\nu}-1}{1-\nu}}\right]^{\rho} \frac{Q(p_j) - Q(p_{j-1})}{g_{j-1}^{\rho} - g_j^{\rho}}, \tag{19}$$

where $Q(p_j) = e^{-\rho\,(Bp_j^{\eta+1} - 1)/(1+\eta)}$, and the choice specific intercept is defined as

$$K_j \equiv \ln\left(\frac{Q(p_j) - Q(p_{j-1})}{g_{j-1}^{\rho} - g_j^{\rho}}\right). \tag{20}$$

Roy's identity implies that the demand function is given by

$$q(p, y) = B\,p^{\eta} y^{\nu}. \tag{21}$$

Note that η is the price elasticity, and ν is the income elasticity. B is the scale parameter of the demand equation. These three parameters are identified since they appear in the demand function.[22]

Proposition 2 implies that ρ cannot be fully identified since this parameter does not appear in the demand function. The only restriction imposed on ρ is that the boundaries evaluated at ρ cannot intersect. Write the boundaries as $\alpha_j(y\,|\,\rho)$ to denote the dependence on ρ. Then vertical product differentiation implies that

$$\alpha_j(y\,|\,\rho_0) < \alpha_{j+1}(y\,|\,\rho_0) \quad j = 1, \ldots, J. \tag{22}$$

As shown in the proof of Proposition 2, for any other ρ that is consistent with these inequality constraints, there exists a distribution of income and tastes that yields observationally equivalent sorting and household demand. The discussion in the previous section directly implies the following proposition:

PROPOSITION 3: *The following results hold for our semiparametric model:*

(i) *The three parameters of the demand equation* (ν_0, η_0, B_0) *are identified from the observed joint distribution of quantity and income given the price variation observed in the market.*

(ii) *For each household type i, we can identify* $J - 1$ *points of* $F_{i0}(\alpha\,|\,y)$ *if we know* ρ_0.

[22] To avoid stochastic singularities, we can easily extend the framework discussed above and assume that the housing demand or expenditures are subject to an idiosyncratic error that is revealed to households after they have chosen the neighborhood. This error term thus enters the housing demand, but does not affect the neighborhood choice. An Appendix that discusses this extension is available upon request from the authors. Alternatively, we can assume in estimation that observed housing demand is subject to measurement error. We follow that approach in our application.

(iii) *The set of ρ's that are consistent with the observed equilibrium outcomes is defined as*

$$\{\rho \,|\, \alpha_{j+1}(y \,|\, \rho) \; > \; \alpha_j(y \,|\, \rho) \quad \forall j\}. \tag{23}$$

This set contains ρ_0.

We can thus either identify or construct bounds for the parameters of interest.

III. Estimation

A. *Measurement Error*

The proofs of identification are constructive and can be used to devise a new estimator for this model.[23] For estimation purposes, it is desirable to relax Assumption 5. The quality of a neighborhood may not be perfectly observed by the econometrician. In most applications, it is likely that we will observe some measures of quality. However, our observed measures may be subject to measurement error. We therefore make the following assumption:

ASSUMPTION 7: *We do not observe* g_j, *but we observe* $\tilde{g}_j$, *which is given by*

$$\tilde{g}_j \; = \; g_j \; + \; \epsilon_j, \tag{24}$$

where ϵ_j *denotes measurement error.*

Our model implies that there exists a monotonically increasing function which maps the price rank of a product into product quality. Let us denote the rank of neighborhood j by r_j. Hence, the ascending bundles property implies that the following equation holds in equilibrium

$$g_j \; = \; g(r_j) \tag{25}$$

for some unknown monotonically increasing function $g(\cdot)$. Substituting equation (25) into equation (24), we obtain

$$\tilde{g}_j \; = \; g(r_j) \; + \; \epsilon_j. \tag{26}$$

Furthermore suppose that $E[\epsilon_j | r_j] = 0$, i.e., the error term in equation (24) is conditionally independent of the rank of the neighborhood. In that case, $g(r_j)$ is

[23] As discussed in Section II, we assume that we have consistent estimators of B, η, and ν. As discussed in Section IV that estimation of these parameters is straight-forward and can be done prior to estimating the other parameters and functions of the model.

nonparametrically identified.[24] We estimate the function $g(r)$ using locally linear kernel regression, as suggested by Jianqing Fan (1992).[25]

The estimation procedure can also be extended to account for multiple product characteristics as long as product quality satisfies an index assumption. Suppose we observe a vector of product or neighborhood characteristics x_j in addition to an education quality measure denoted by e_j. Assume that consumer preferences only depend on the linear index $g_j = e_j + x_j'\gamma$. Since neighborhood quality can be measured in arbitrary units, we have normalized the coefficient of one of the components in the index (education) to be equal to one to achieve identification. Substituting the index equation above into equation (26) and rearranging terms yields

$$e_j = g(r_j) - x_j'\gamma + \epsilon_j. \tag{27}$$

This approach then gives rise to a model that is formally equivalent to the one studied and estimated in Peter M. Robinson (1988). Note that the coefficient γ measures the impact of other characteristics relative to education. We implement this estimator in the following application. Note that we can also interpret ϵ_j as an unobserved product characteristic.

B. *Imposing the Shape Restrictions*

For the model to be well defined, we also need the neighborhood specific intercepts to be monotonically increasing: $K_1 < \cdots < K_J$. A necessary, but not sufficient condition, for this to hold is that $g(r)$ is monotonically increasing in r. If $g(r)$ has a sufficient degree of curvature, i.e., if the differences in neighborhood quality are sufficiently large relative to the differences in observed prices, then the intercepts are also monotonically increasing functions. It is, therefore, desirable to impose these shape restrictions in estimation. Moreover, by testing whether these curvature restrictions hold in the data, we can determine which values of ρ are admissible.

We impose these curvature restrictions using the isotone-kernel regression estimator proposed by Enno Mammen (1991). This estimator uses a two step procedure to recover the monotonic function. First, a nonparametric estimator $\hat{g}(\cdot)$ is obtained, using local linear kernel regression. This function may not be monotonically increasing. In the second step, the estimated function is projected onto the space of shape-restricted (e.g., monotonic) functions. The estimator is defined by

$$g^{sr}(r) = \arg\min_{g \in G} \int (g(r) - \hat{g}(r))^2 \, dr, \tag{28}$$

[24] Instead of estimating g as function of rank, one also estimates $\tilde{g}_j = g(p_j) + \epsilon_j$. In our application, we find that the two different approaches yield similar results.

[25] We have not fully worked out a precise limiting argument for our model. One can view our model as a discretized approximation of a hedonic model with a continuum of choices. In the limiting case, public good provision would be a function of the percentile rank of the community. We conjecture that there exists an economy with a continuum of choices that is the limit of these economies with finite, discrete choices. However, we have not proved this result. As the sample size increases, there will be more communities with similar characteristics. One would then need to show that the standard regularity conditions for nonparametric estimators are satisfied. For an overview of nonparametric techniques see, for example, Adrian Pagan and Aman Ullah (1999).

where G denotes the class of shape-restricted functions.

We define $m = -g^{\rho}$ and $\hat{m} = -\hat{g}^{\rho}$. The curvature restrictions then imply that $m(r)$ is sufficiently monotonically increasing in r. Our constrained estimator of the function is then obtained by minimizing the following objective function:

$$\min \sum_{j=1}^{J} (m_j - \hat{m}_j)^2, \tag{29}$$

subject to the constraints that

$$m_{j-1} \leq m_j - \delta_1 \tag{30}$$

$$\frac{1}{Q_{j-1} - Q_{j-2}}(m_{j-2} - m_{j-1}) \leq \frac{1}{Q_j - Q_{j-1}}(m_{j-1} - m_j) - \delta_2$$

for nonstochastic constants $\delta_1, \delta_2 > 0$. Mammen (1991) shows that a similar estimator is consistent and derives rates of convergence.

We implement this estimator using quadratic programming techniques. Note that this estimator depends on ρ since the restrictions depend on ρ. Our procedure for recovering the identified set requires estimating the shape-restricted function on a grid of points for ρ.[26]

C. *Testing the Shape Restrictions and Bounding ρ*

Proposition 3 shows that there exists a set of ρ's that are consistent with the ascending bundles property. Once we allow for measurement error in quality, we need to test whether the shape restrictions are valid given a parameter ρ. In this section, we discuss how to test these shape restrictions and construct bounds for ρ. Our test procedure follows the basic ideas outlined in Hall and Adonis Yatchew (2005).

To develop our test, let us normalize the rank of a community to be between 0 and 1, i.e., $r_i = i/J$. Note that the rank is, by construction, uniformly distributed. Let $A \subset [0,1]$ denote the area of integration. For example, $A = [0,a] \cup [b,1]$ with $a < b$ would cover the "tail" regions where we expect violations of the null. Consider the test statistic

$$T_A = \int_A (g(r) - g^{sr}(r))^2 \, dr, \tag{31}$$

where $g^{sr}(r)$ is the function in the shape restricted class that is closest to $g(r)$ under the L^2 norm. A feasible test statistic is then given by

$$\hat{T}_A = \sum_{i, r_i \in A} (\hat{g}(r_i) - \hat{g}^{sr}(r_i))^2. \tag{32}$$

[26] Standard errors and confidence bands can be computed using bootstrap techniques. Bootstrap techniques are discussed in Bradley Efron and Robert J. Tibshirani (1993) and Hall (1994).

As pointed out by Hall and Yatchew (2005), we have strong reasons to believe that under the null $\sqrt{J}\,\hat{T}_A \to V$, where V is a continuous random variable. However, deriving the asymptotic distribution of the test statistic is difficult.

Hall and Yatchew (2005) suggest employing the bootstrap to compute the critical values for this test statistic. To use the bootstrap procedure, we need to construct the distribution of $\hat{T}_A$ under the null hypothesis. We cannot sample from the underlying empirical distribution of the data since we do not know whether the empirical distribution satisfies the null hypothesis. Hence, we need to devise a bootstrap sampling algorithm which imposes the null on the data generating process.

To see how that can be done, consider the shape restricted estimator and define the residuals of the restricted model to be

$$\hat{\epsilon}_i^{sr} = \tilde{g}_i - \hat{g}^{sr}(r_i). \tag{33}$$

We recenter the residuals by subtracting the sample mean of the residuals from the individual residuals. The bootstrap then samples from the recentered distribution of the $\hat{\epsilon}_i^{sr}$. Let $\epsilon_1^s, \ldots, \epsilon_N^s$ denote a bootstrap sample and define

$$g_i^s = \hat{g}^{sr}(i/J) + \epsilon_i^s. \tag{34}$$

For each bootstrap sample s, we then compute the unconstrained and the constrained estimator, and thus evaluate the test statistic $\hat{T}_A^s$.

By repeating this procedure S times, we can trace out the distribution of the test statistic under the null hypothesis

$$\Pr\{\hat{T}_A \leq x\} \approx \frac{1}{S}\sum_{s=1}^{S} 1\{\hat{T}_A^s \leq x\}. \tag{35}$$

It is straight-forward to read off the critical values of our test statistic from the distribution above. We compare the realization of our test statistic in equation (31) with the critical value and determine whether or not to reject the null.

To determine which $\rho's$ are feasible, we pick a grid of values for ρ. We then implement the test procedure for each value of ρ and determine the set of ρ's that are consistent with the ascending bundles property.

D. *Estimating the Conditional Distribution of Tastes*

The constrained estimator of $g(\cdot)$ directly implies an estimator of the boundary indifference loci $\alpha_j(y\,|\,\rho)$ which are well behaved. We denote these estimators by $\hat{\alpha}_j(y\,|\,\rho)$. Note that the constrained estimator of the function $g(\cdot)$ depends on ρ since the constraints are functions of ρ. As a consequence $\hat{\alpha}_j(y\,|\,\rho)$ also depends on ρ.

We observe the empirical market shares and income distributions for each neighborhood, $\{n_{ij}^N, f_{ij}^N(y)\}_{j=1}^J$, where N denotes the relevant sample size. Following the discussion of identification in Section II, a nonparametric estimator of the $J - 1$

points of the conditional distribution of tastes given income, $\hat{F}_i^N(\alpha \,|\, y)$, is then given by

$$\hat{F}_i^N(\hat{\alpha}_j(y \,|\, \rho) \,|\, y) = \sum_{k=1}^{j} \frac{f_{ik}^N(y)}{f_i^N(y)} n_{ik}^N \quad j = 1, \ldots, J - 1, \tag{36}$$

where $f_i^N(y)$ denotes density that corresponds to the empirical income distribution of type i households in the market. It is straightforward to show that for any j,

$$\sqrt{hN}\left(\sum_{k=1}^{j} \frac{f_{ik}^N(y)\; n_{ik}^N}{f_i^N(y)} - \sum_{k=1}^{j} \frac{f_{ik}(y)\; n_{ik}}{f_i(y)}\right) \xrightarrow{d} N(0, \sigma_j^2(y)), \tag{37}$$

where the asymptotic variance $\sigma_j^2(y)$ can be computed from the variances of the density estimators using the delta-method. This estimator depends on ρ. Thus, for any admissible value of ρ we obtain a different conditional distribution of tastes.

IV. Data

Our application focuses on residential choices and housing demand in Allegheny County, which includes Pittsburgh as its central city. Allegheny County consists of 130 municipalities. Since Pittsburgh is large, both in land area and population, we divide Pittsburgh based on its 32 wards. After combining some small neighborhoods, we obtain a sample with 150 communities.

A. *Housing Demand*

We have obtained a detailed dataset of all residential housing units in Allegheny County. This dataset contains housing values and housing characteristics for essentially all residential properties in Allegheny County. Our dataset consists of 93,763 properties which were recently sold. The dataset contains a detailed list of housing characteristics including grade and condition assigned by an assessor of the property, year built, type of residence, finished living area, total number of rooms, number of bedrooms, number of full bathrooms, number of half bathrooms, whether the residence has a fireplace, and whether the residence has central air conditioning.

Housing values, v_{jn}, are converted into imputed rents, r_{jn}, using the formula suggested by James M. Poterba (1992),

$$r_{jn} = [(1 - \tau_y)(i + \tau_j^p) + \beta + m + \delta - \pi]\, v_{jn} \tag{38}$$

with average income tax rate $\tau_y = 0.15$, nominal interest rate $i = 0.079$, risk premium $\beta = 0.04$, maintenance minus depreciation $m - \delta = 0.02$, inflation rate $\pi = 0.0286$, and community specific property tax τ_j^p.

Some communities in Allegheny County also rely on local income taxes. We convert local income taxes into implied property tax rates. Let τ_j^p be the property tax rate and τ_j^y be the local income tax rate in community j. We compute a property tax rate τ_j that yields the same revenue from the mean household in each community:

$$\tau_j = \tau_j^p + \tau_j^y \frac{\overline{y}_j}{\overline{v}_j}. \tag{39}$$

To obtain housing prices, we assume that the imputed rent satisfies $r_{jn} = p_j h_{jn}$ and that $\ln(h_{jn}) = \delta z_{jn} + \epsilon_{jn}$, where z_{jn} are the observed characteristics of house n in community j. We then estimate a hedonic regression using the micro level housing data:

$$\ln r_{jn} = \sum_{j=1}^{J} d_{jn} \ln p_j + \delta z_{jn} + \epsilon_{jn}. \tag{40}$$

Here, d_{jn} is equal to one if house n is located in community j, and zero otherwise. The community specific intercepts of the hedonic regression with fixed effects can be interpreted as housing price estimates as discussed in detail in Sieg et al. (2002). When decomposing rents into a price and a quantity component, we cannot separately identify the price per unit and the intercept of quantity index. Units of measurement for housing services are inherently unobserved. We can normalize the price of housing in the lowest ranked community to be equal to one. This normalization then implicitly determines the units of measurement for housing services.[27]

The R^2 for the housing price regression is 0.5. Hence, we control for much of the differences in the quality of housing across communities. Housing prices after taxes range from 1.06 to 6.96.

After estimating housing prices, we estimate the parameters of the housing demand equation using aggregate census data. Our model implies that

$$\ln r_{jq} - \ln p_j = \ln \beta + \nu \ln y_q + \eta \ln p_j + \epsilon_{jq}^h, \tag{41}$$

where ϵ_{jq}^h denotes the error term in the model which may be due to measurement error in housing expenditures. r_{jq} represents the qth housing quantile, and y_{jq} represents the respective income quantile. We use the 10 percent through the 90 percent deciles in our estimation. We estimate the system of equations above using seemingly unrelated regressions. The estimation results for the housing demand model yield an estimate for ν of 0.784 (0.017). The price elasticity η is estimated at -0.514 (0.027), and the intercept B is equal to 1.161 (0.182).[28]

To estimate the housing demand parameters, it might be preferable to use an instrumental variable estimator. If we view the errors in the housing expenditure function as demand shocks, housing prices will be correlated with the error terms. In that case, we could use housing supply shifters to generate instruments for housing prices in the demand equation. If we view the error terms as measurement error in the dependent variable, there is no obvious need for instruments. Since the focus

[27] Epple, Gordon, and Sieg (2010b) provide an alternative approach for estimating the production function for housing which also deals with the fact that price per unit of housing services and quantities for housing services are inherently unobserved by the econometrician.

[28] These estimates are plausible and correspond to previous estimates reviewed in Sieg et al. (2004).

of this paper is on the semi-parametric part of the analysis, we have followed the simpler measurement approach and used seemingly unrelated regressions instead.

B. *Neighborhood Amenities and Demographic Data*

We include three measures of local amenities in our index of public good provision: education quality, crime, and travel time to the city center. We construct an education index based on the PSSA, a math and reading test administered in all public schools for grades 5, 8, and 11 in Pennsylvania in the school year 1999–2000. Data on participation rates and average scores are available for each of the six tests for school districts and individual schools. There are no missing observations and participation rates are high. We average the six scores, weighted by enrollment in the different grades. For municipalities outside of Pittsburgh, a single school district sometimes serves several municipalities. We assign each of these municipalities the score for the school district. Getting education scores for the wards within the city of Pittsburgh is considerably more difficult. Here, we rely on data reported by individual schools. School attendance zones for elementary schools, middle schools, and high schools sometimes overlap with the boundaries of the wards. In these cases, we average the scores of all schools serving a ward weighted by the fraction of households served by that school.

To construct a crime index, we rely on two data sources: the Uniform Crime Report and data collected by the *Pittsburgh Post-Gazette*. The Uniform Crime Report is a yearly survey of the number and types of crime in each municipality in the United States. It reports the number of actual incidents as reported by the police for murder, rape, robbery, assault, burglary, larceny, theft, as well as other crimes. To obtain crime rates from within the city's 32 wards, we rely on data reported by the *Pittsburgh Post-Gazette*. We adjust these numbers by a multiplicative factor such that the numbers reported by the *Post-Gazette* for Pittsburgh as a whole match the numbers in the Uniform Crime Report. Crime is measured in incidents per 100,000 individuals. The rush hour travel time to the central city of each municipality outside of Pittsburgh is taken from a dataset provided by the Southwestern Pennsylvania Commission. Travel time is measured in minutes of driving time.

Demographic, income, housing, and rental data are based on the US census. The data are aggregated from census tracts up to the wards and municipalities that form the communities in our dataset. Summary statistics of the sample of communities are reported in Table 1.

V. Empirical Results

We estimate the function $g(r)$ using locally linear kernel regression. The results of the estimation are plotted in Figure 1. We find that the smooth estimator $\hat{g}(r)$ does not violate the monotonicity condition. However, the unconstrained estimate of the function does not have enough curvature to ensure that the neighborhood specific intercepts are increasing.[29]

[29] We also control for differences in crime and commuting time to the city center in estimation using a partially linear estimator suggested by Robinson (1988). The point estimate of the coefficient of crime is 0.0060 (0.0023),

Table 1—Descriptive Statistics

Variable	Mean	SD	Minimum	Maximum
Population	8,539	8,551	467	46,809
Number of households	3,581	3,538	204	19,467
Percent with children	0.2665	0.0718	0.0632	0.5145
Price (before taxes)	2.9527	0.9403	1.0000	6.7302
Price (after taxes)	3.0919	0.9685	1.0638	6.9637
Education index	1.2944	0.0836	1.0917	1.4699
Total crime index	690	888	0	8,197
Property tax rate	0.0202	0.0026	0.0140	0.0283
Rush hour travel time	24.22	11.15	1	57
Income tax rate	0.0426	0.0077	0.0380	0.0568
Imputed total tax rate	0.0490	0.0094	0.0319	0.0784
Mean income	52,947	30,350	19,580	233,674
Mean housing value	100,250	72,587	26,658	519,080
Mean rent	414	149	209	1,156

Notes: Education is measured in PSSA scores. Crime is measured in incidents per 100,000. Travel time is measured in driving minutes.

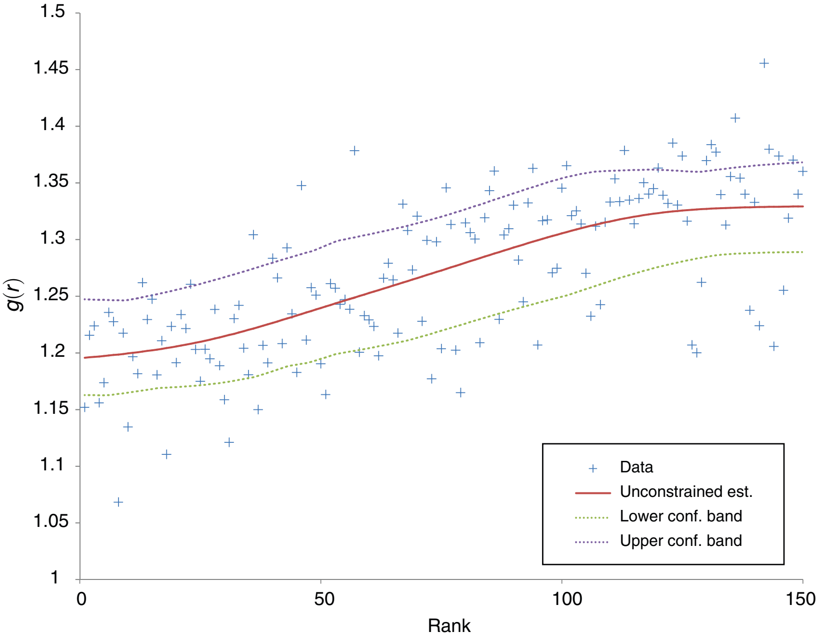

Figure 1. Estimation of $g(r)$

We, therefore, impose the curvature restriction implied by the vertical differentiation property and implement the restricted estimator of the $g(\cdot)$ function. Since the constraints depend on the value of ρ, we estimate a number of constrained functions using values of ρ ranging from -1.2 to -0.1.[30] The results of these different estimators are plotted in Figure 2.

and the coefficient for travel time is -0.0024 (0.0004).

[30] Using the Epple and Sieg (1999) parametric framework, we obtain a point estimate of -0.198.

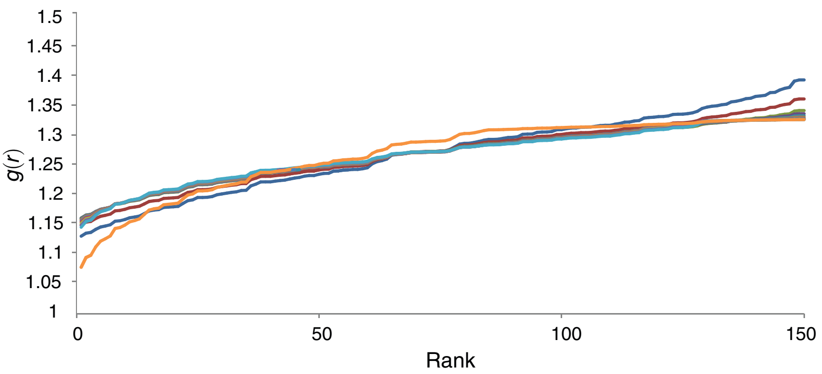

Figure 2. Constrained Estimation of $g(r)$

Note: The figure plots the constrained function for ρ ranging between -1.2 and -0.1.

We find that the constrained functions have similar shapes for values of ρ ranging approximately from -0.2 to -0.8. As one goes outside this interval, we find that we need more curvature, especially in the tails of the function to accommodate the shape restrictions.

Next, we implement our testing procedure to determine the admissible set for ρ as described in detail in Section IVC. For each value of ρ, we plot the p-value associated with the Hall and Yatchew (2005) test statistic in Figure 3. The Hall and Yatchew (2005) test works fairly well in our application. Using a 95 percent level test, we find that the admissible set of ρ is given by the interval $[-1.16, -0.14]$. These results are qualitatively and quantitatively consistent with the graphical analysis in Figures 1 and 2. Once we move outside this interval, the restricted function significantly differs from the unrestricted function, which suggests that the model violates the shape restrictions imposed by the ascending bundles property.[31]

Given the admissible set of ρ's, we then estimate the conditional distribution of tastes for quality. In our application, we observe the sorting by households across communities. In our sample, 26.7 percent of the households living in Allegheny County have children. The average income of these households is \$66,858 with a standard deviation of \$67,655. Households without children have an average income of \$47,803 with a standard deviation of \$53,056. To characterize the observed sorting of household types across communities, we compute the following cumulative probabilities:

$$\sum_{k=1}^{j} \frac{f_{ik}^N(y)}{f_i^N(y)} \, n_{ik}^N .$$

Recall that these probabilities measure the share of type i households with income level y that live in communities which have housing prices less than or equal to the price of community j.

[31] We also implemented a second test that is based on an equally spaced grid of points. The alternative procedure produced qualitatively similar results, and indicated that the identified set is $[-0.78, -0.16]$.

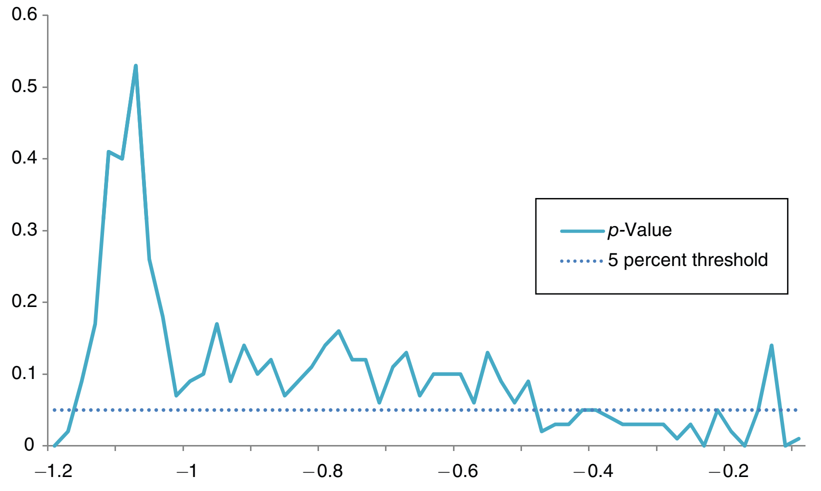

Figure 3. Testing for Admissible Values of ρ

Table 2—Household Sorting

	Households with children				Households without children			
Housing price	Income				Income			
Percentile	19,330	37,921	66,247	102,239	19,330	37,921	66,247	102,239
10	0.232	0.090	0.044	0.014	0.110	0.059	0.051	0.030
	(0.006)	(0.003)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.002)
20	0.339	0.155	0.080	0.036	0.194	0.128	0.101	0.064
	(0.007)	(0.005)	(0.002)	(0.002)	(0.003)	(0.002)	(0.001)	(0.002)
30	0.467	0.258	0.149	0.060	0.293	0.215	0.176	0.100
	(0.009)	(0.005)	(0.003)	(0.002)	(0.003)	(0.003)	(0.002)	(0.003)
40	0.628	0.448	0.286	0.138	0.451	0.372	0.327	0.206
	(0.012)	(0.007)	(0.004)	(0.004)	(0.003)	(0.004)	(0.003)	(0.004)
50	0.704	0.546	0.375	0.219	0.517	0.453	0.414	0.277
	(0.012)	(0.007)	(0.005)	(0.005)	(0.003)	(0.005)	(0.003)	(0.005)
60	0.780	0.656	0.494	0.354	0.642	0.572	0.527	0.399
	(0.013)	(0.007)	(0.005)	(0.006)	(0.003)	(0.005)	(0.004)	(0.004)
70	0.837	0.744	0.587	0.433	0.710	0.656	0.612	0.480
	(0.014)	(0.007)	(0.005)	(0.007)	(0.003)	(0.005)	(0.004)	(0.005)
80	0.901	0.863	0.776	0.622	0.827	0.791	0.764	0.650
	(0.015)	(0.009)	(0.006)	(0.008)	(0.004)	(0.006)	(0.005)	(0.006)
90	0.958	0.956	0.933	0.857	0.913	0.918	0.900	0.834
	(0.015)	(0.009)	(0.007)	(0.009)	(0.004)	(0.006)	(0.005)	(0.007)

Table 2 reports these estimated cumulative probabilities for households with and without children, as well as estimated standard errors. We estimate these probabilities for four different income levels: \$19,330, \$37,921, \$66,247, and \$102,239. These income levels correspond to the twenty-fifth, fiftieth, seventy-fifth, and ninetieth percentile of the income distribution in the metropolitan area. We estimate these measures for each decile of the housing price distribution. Each decile corresponds to 15 communities in our sample.

Table 2 provides some interesting new insights. Consider low-income households with annual income of \$19,330. Twenty-three percent of households with children live in the 15 cheapest communities, i.e., lowest decile of communities. Only 11 percent of households without children live in these communities. The opposite pattern holds for richer households with annual income of \$102,239. Fifty-seven percent of households with children live in the 45 most expensive communities. Here, the corresponding number for households without children is 52 percent. Given the estimated standard errors, these differences are statistically significant. We thus conclude that the sorting of households with children exhibits more stratification by income than the observed sorting of households without children. Households with children and income levels below the median metropolitan income are more likely to live in cheaper communities than households without children. The opposite is true for households with high levels of incomes. High-income households with children have stronger tastes for high price (and high amenity) communities than households without children.

For any value of ρ that is admissible, we then compute the boundaries between adjacent communities. Given the parametrization of our utility function, these boundaries are given by the following expression:

$$\ln(\alpha_j) = K_j + \rho \frac{y^{1-\nu} - 1}{1 - \nu}. \tag{42}$$

Given these boundaries and the results in Proposition 3, it is straight-forward to construct the conditional distribution of tastes given income. To illustrate the algorithm, we set $\rho = 0.52$, which is the midpoint of the admissible interval of ρ based on the Hall Yatchew Test. In Figure 4, we plot the distribution for tastes conditional on two income levels (\$37,921 and \$66,247). The solid (dashed) line denotes households without (with) children. We find that higher income households have, on average, significantly lower tastes for local public goods than lower income households. This result is true for households with and without children. Lower income families with children tend to have lower tastes for public goods than households without children. We also computed the conditional distribution of tastes for other feasible values of ρ and the results were qualitatively the same.

To compare our results to parametric techniques, we implemented parametric estimators along the lines developed by Epple and Sieg (1999). The parametric estimate of ρ is approximately -0.2, which is close to our semiparametric lower bound. The parameter estimates for the index of public goods are similar with crime and driving distance having negative coefficients. The main difference is the fit of the model. The semiparametric model fits the observed income distributions perfectly, while the parametric model yields an imperfect fit.

VI. Conclusions

We have discussed nonparametric and semiparametric identification of locational equilibrium models. Our results show that the model considered in this paper is partially nonparametrically identified. The proofs of (partial) identification are

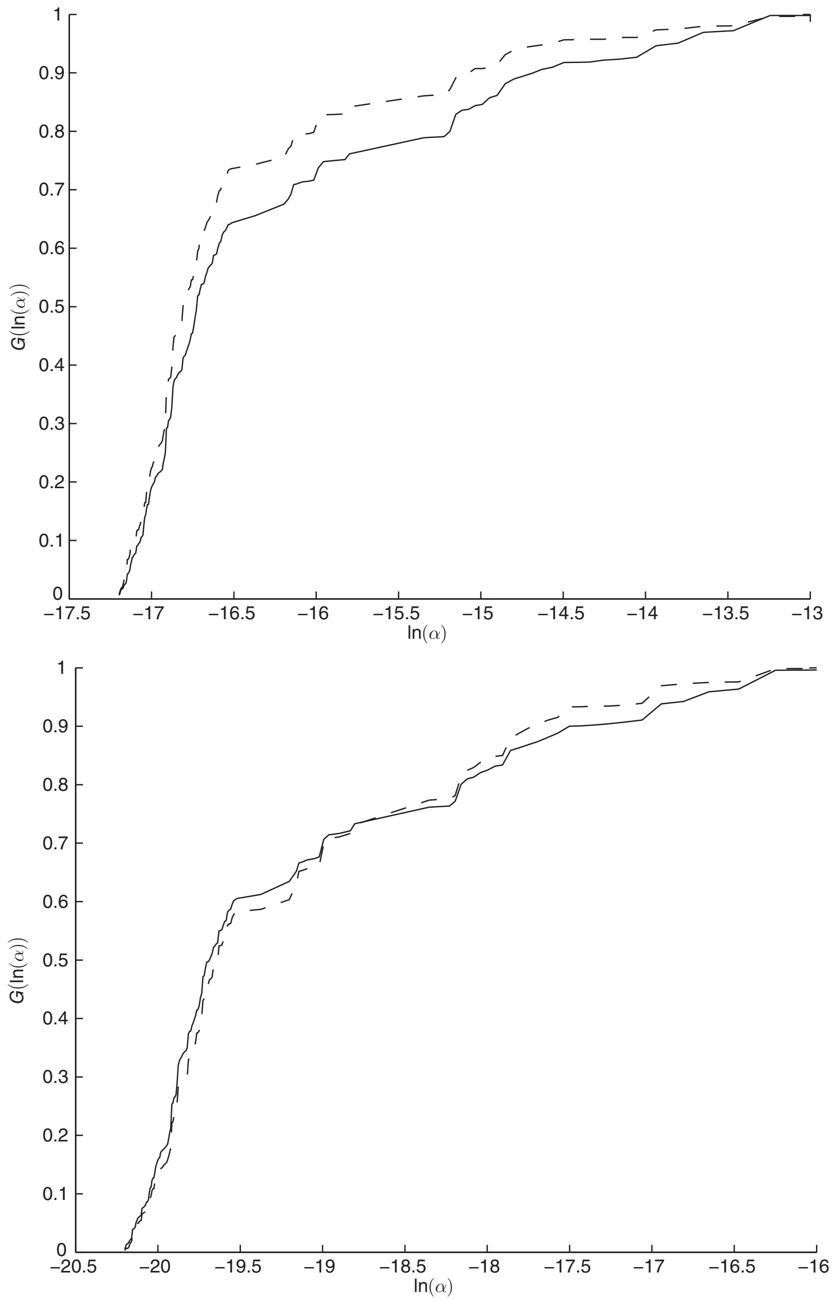

Figure 4. Conditional Distribution of Tastes

Notes: The top (bottom) panel shows distribution conditional on income equal \$37,921 (\$66,247). The solid (dashed) line denotes households without (with) children.

constructive. We have shown how to derive a new two-step estimator for the semiparametric model. This estimator differs significantly from previously used parametric estimators that are based on share inversion algorithms. We have discussed the properties of the new estimator and provided some simple algorithms that can be used to implement the estimator. We have studied residential sorting and housing demand in a new application that focuses on sorting of households with and without children across municipalities in Allegheny County. We have documented the observed sorting of each household type by income among the set of communities in our sample. Our empirical findings suggest that there are significant differences in the observed sorting patterns of household types. We find that households with children seem to be more sensitive to differences in housing prices and local public goods than households without children. The sorting pattern of households with children exhibits a lot more stratification by income than the corresponding pattern for households without children. Moreover, we find that low-income households with children have, on average, lower tastes for public goods than households without children. The opposite is true for households with higher income levels.

We view the findings of this paper as encouraging for further research in this area. There seems to be ample scope for using nonparametric and semiparametric estimation techniques to estimate richer specification of the type of locational equilibrium models considered in this paper. We have limited our discussion to hierarchical models in which there exists a clear ranking among the set of communities. If households have heterogeneous tastes defined over a vector of local public goods and amenities, household-sorting equilibria do not necessarily satisfy the ascending bundles property. Moreover, there will be both vertical and horizontal product differentiation in equilibrium. Alternatively, one could consider extensions of the hierarchical model which allow for differences in utility functions across types or additional unobserved heterogeneity in tastes for housing. For low-income households with children, providing basic necessities, such as food and shelter, may take precedence over concerns for local public goods. Establishing conditions for non- or semiparametric identification and developing feasible estimators for these models is an important area for future research.

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