# Assignment 2

1. Program up the secant method. Recall that the secant method is defined by the iteration sequence, $x_k = x_{k-1} - \frac{f(x_{k-1})(x_{k-1} - x_{k-2})}{f(x_{k-1}) - f(x_{k-2})}$. Apply it to find the solution to $e^x + 0.3x - 0.7 = 0$.

2. Consider the function $f(x) = (0.3x^2 - 0.6x)e^x + 0.2$ and suppose that we would like to find solutions to $f(x) = 0$.

(a) Plot $f(x)$.

(b) Use the bisection method to find all solutions to $f(x) = 0$

(c) To apply Newton's method, you will have to take the derivative of $f(x) = (0.3x^2 - 0.6x)e^x + 0.2$ by hand and program it up. Program up a function for $f'(x)$ and then check that you computed $f'(x)$ correctly by comparing your program to numerical derivatives for various values of $x$

(d) Use Newton's method, Newton's method with numerical derivatives, and the secant method to find solutions to $f(x) = 0$. Experiment with different starting values and comment on the performance of the algorithm when different starting values are used.

3. Suppose that $X_n \sim N(\mu, \sigma^2)$. We can show that the log-likelihood for $(\mu, \sigma^2)$ is given by, $\sum_{n=1}^{N} \log \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(X_n-\mu)^2/\sigma^2} = -\frac{1}{2} N \log(2\pi) - N \log \sigma - \frac{1}{2\sigma^2} \sum_{n=1}^{N} (X_n - \mu)^2$.

(a) Generate a random vector of data with $N = 1000$ data points from this model assuming that the true parameters are $(\mu_0, \sigma_0^2) = (-0.5, 1.7)$

(b) Program up the likelihood for this model

(c) Maximize the likelihood using the BFGS method. Check to make sure that the solution you found is a local maximum. (Note that the correct solution can be computed as $\hat{\mu} = \bar{X}$ and $\hat{\sigma}^2 = \frac{N-1}{N} s^2$).

(d) Maximize the likelihood using the Nelder-Mead method. Again check to make sure that the solution you found is a local maximum

(e) Program up the gradient function for the likelihood. Check that you programmed up the gradient correctly by comparing it to the numerical derivatives

(f) Maximize the likelihood using the BFGS method with analytical gradient. Again check to make sure that the solution you found is a local maximum